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# Simple Heuristics as Equilibrium Strategies in Mutual Sequential Mate Search\*

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**Abstract.** In this paper, we study whether simple heuristics can arise as equilibrium strategies in mutual sequential mate search. To this aim, we extend the mate search model of Todd and Miller (1999), involving an adolescence (learning) phase followed by an actual mating phase, to a strategic game where the players, as the individuals in the mating population, choose before starting the adolescence phase, the best rule - among the four available search (aspiration adjustment) rules - to maximize their likelihood of mating, given the choice of other individuals. Conducting Monte Carlo simulations, we show that the use of the Take the Next Best Rule by the whole population never becomes a (Nash) equilibrium in the simulation range of adolescence lengths. While the unanimous use of the Adjust Relative Rule by the whole population arises as an equilibrium for a wide part of the simulation range, especially for medium to high adolescence lengths, the rules Adjust Up/Down and Adjust Relative/2 are unanimously chosen as equilibrium strategies for a small part of the simulation range and only when the adolescence is long and short, respectively.

**Key words:** Mate Choice, Mate Search, Simple Heuristics, Agent-Based Simulation, Stability, Equilibrium Strategies

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# 1 Introduction

It has been fifty years since the problem of mate search was studied by Gale and Shapley (1962). Their seminal work offered an iterative algorithm, called the deferred acceptance algorithm, under which a population of males and females has always a stable matching where there exist no two agents of opposite sexes who are not a pair but prefer each other to their current partners, and no individual who is matched but prefer being single to his/her partner. This algorithm, which was independently discovered by the National Residency Matching Program (NRMP) in the United States (US) and had been used since 1950s in matching medical interns with hospital residency positions (as shown by Roth, 1984), gained its popularity especially with its use to match students with public high schools in New York City and Boston.<sup>2</sup>

While the two-sided stable matching model of Gale and Shapley (1962) has led to the emergence of a large literature in economic theory and applied mechanism design, the amount of research in this literature studying the formation of marriages in societies is extremely little.<sup>3</sup> One reason is that in marriage environments, unlike in school choice or hospital-intern problems, there exists no central agency applying a particular matching algorithm. Besides, individuals have no information about potential mates before the actual matching takes place. Therefore, it has become inevitable to study the formation of marriages using ‘decentralized’ and ‘sequential’ models of mate search, where individuals gain, by sequentially encountering some potential mates, all relevant information on which they base their final mating decisions.<sup>4</sup> Relatedly, a strand of literature (Dombrovsky and Perrin, 1994; Mazalov et

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<sup>2</sup>See Abdulkadiroğlu and Sönmez (2003) for a pioneering work, and Abdulkadiroğlu (2013) and Pathak (2011) for surveys, on school choice, and Roth and Sotomayor (1990) for a wide range of earlier results in stable matching theory.

<sup>3</sup>Stable matching theory was applied to study the formation/dissolution of marriages only very recently by Mumcu and Saglam (2008) when utilities are transferable between mates and by Saglam (2011) under nontransferable utilities.

<sup>4</sup>See Kalick and Hamilton (1986) for an early example of computer-based, decentralized and

al., 1996; Todd and Miller, 1999; and Collins et al., 2006) conditioned the information for mating decisions on the self mate values of individuals, also assuming that individuals do not completely know but can partially learn (or approximate) their self mate values by using feedbacks from potential mates they interact before they are mated. Among this literature, the paper of Todd and Miller (1999) was the first to consider a mate-choice model with two-sided (mutual) search strategies. In this model, individuals first go through an ‘adolescence’ (learning) phase in which they randomly interact (date) with a number of individuals of the opposite sex and possibly exchange information with their dates. After each interaction each individual adjusts his/her aspiration level (as a proxy of his/her self mate value) according to a particular adjustment rule assumed to be used by the whole population. Individuals next proceed to a ‘mating’ phase where they randomly interact with potential mates and decide whom to make a proposal for mating. In this phase, each pair of individuals in the mating pool are considered to be successful and removed from the pool as mated if they simultaneously make proposals to each other. This phase ends after a stage at which either the mating pool becomes empty or each individual in the mating pool has already been paired unsuccessfully with all available individuals of the opposite sex.

The adjustment rules considered by Todd and Miller (1999) in the adolescence phase of their model involve Take the Next Best, Adjust Up/Down, Adjust Relative, and Adjust Relative/2.<sup>5</sup> According to the Take the Next Best Rule, individuals start the adolescence period with an initial aspiration level of zero, and at each instance

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sequential, models of mate search.

<sup>5</sup>The adjustment rules considered by Todd and Miller (1999) also involve the Mate Value -  $\alpha$  Rule, according to which the aspiration level of each individual is constant over the adolescence period and formed by subtracting a prescribed constant  $\alpha$  (set to 5 in their simulations) from one’s self mate value. We have chosen to exclude this rule from the scope of our paper since it requires, as already noted by Todd and Miller (1999), that each individual knows his/her self mate value, a highly unrealistic assumption.

of dating each individual sets his/her aspiration to the mate value of his/her date if that value is above his/her current aspiration level. Thus, individuals leave the adolescence phase with an aspiration level set to the highest mate value they have observed.

The remaining three adjustment rules set the initial aspiration level of each individual to the average mate value of all individuals of the same sex. These rules also require that each individual exchanges information with his/her date as to whether they have found each other desirable; i.e., the observed mate value of the date is above one's aspiration level. According to the Adjust Up/Down Rule, each individual adjusts, at each instance of dating, his/her aspiration upwards by a constant shift parameter if he/she learns that the date finds him/her desirable, and adjusts his/her aspiration downwards by the same parameter otherwise. The Adjust Relative Rule differs from the previous rule in that if the date's mate value is above the current aspiration level of an individual and the date still finds this individual desirable, the individual raises his/her aspiration level. Conversely, if the date's mate value is below the current aspiration level of an individual and the date does not find this individual desirable, the individual reduces his/her aspiration level. In other possible cases, individuals do not make any adjustments. In Adjust Up/Down Rule and Adjust Relative Rule, the adjustment parameter is constant during the adolescence phase and inversely related to the length of this phase (i.e., the common number of dates interacted by each individual). Finally, the Adjust Relative/2 Rule differs from Adjust Relative in the adjustment parameter, which is no longer constant during the adolescence phase but is dependent on the difference between the aspiration level of each individual and the mate value of his/her date.

Computer simulations of Todd and Miller (1999) show that among the four adjustment rules the TNB rule yields the lowest number of matings. The highest number of matings are generated by the Adjust Relative/2 Rule when adolescence length is short to medium, and by the Adjust Up/Down and Adjust Relative Rules when adolescence is longer. Since in terms of the likelihood of mating no adjustment

rule dominates every other rule for all adolescence lengths, one needs to consider additional measures of ‘population-level mating success’ to identify the best adjustment rule. One such measure, Todd and Miller (1999) use in their study, is the mean mate value of all mated individuals’, with middle values indicating more successful mate search strategies. Another measure they consider is the mean within-pair difference in mate value, with lower values indicating strategies that are more successful. Todd and Miller (1999) show that of the three rules that all dominate the TNB rule in terms of the likelihood of mating, Adjust Relative/2 has a better performance in terms of these additional measures of success, than the other two rules, namely Adjust Up/Down and Adjust Relative, for almost all adolescence lengths.

Clearly, the (ex-post) instability of some matings is inevitable in environments where matings are decentralized, individuals have incomplete information about potential mates and the search is not exhaustive. The mean difference between the mate values of partners, as a measure of mating success, can provide some indirect information about the stability of the mated pairs formed under a particular adjustment rule. As already argued by Todd and Miller (1999), with higher values of this measure an adjustment rule may lead to less stable matings, since mated pairs with diverse mate values are more prone to the danger of partner switching in a dynamic framework. Recently, Eriksson and Hägsström (2008) has dealt with directly estimating the degree of instability that one can expect in decentralized matching environments. Using the proportion of blocking pairs among all possible pairs as a measure of instability, they show that in environments where all individuals use a particular heuristic with a threshold lowered gradually over the mate search (as in Simão and Todd, 2002), the expected instability of matchings tends to zero as the number of agents grows if individuals’ preferences are random and independent. Following up this work, Eriksson and Strimling (2009) show, with the help of experimental data, how the total search effort and the expected instability of the matching outcome vary with various other preference structures.

Inspired by the previous works studying the stability of matching outcomes under

simple heuristics in mate search, we would like to ask in this study an entirely new, yet complementary, question: whether the simple heuristics/rules used in the mate search are themselves stable when individuals can act strategically,<sup>6</sup> i.e., whether all individuals using a particular adjustment rule can be an equilibrium à la Nash (1950), of a normal-form game where the set of players involve all individuals in the population, the strategies of the players are the adjustment rules described above and the payoff of each player at each strategy profile is his/her likelihood of mating.

## 2 Mutual Sequential Mate Search Model

We consider the mutual sequential mate search model of Todd and Miller (1999), where a population  $N$  involves a set of males,  $M = \{m_1, m_2, \dots, m_n\}$  and a set of females  $F = \{f_1, f_2, \dots, f_n\}$ , with  $n > 1$ . Each individual  $i \in N$  has a (self) mate value,  $v(i)$ , which is a randomly drawn from the uniformly distributed values over the interval  $[0, V]$ . Mate value of each individual is always unknown to himself/herself.

Mate search consists of two phases. The first phase is called ‘adolescence’ or ‘learning’ phase, where each individual adjusts his/her aspiration level based upon the adjustment rule he/she follows. This phase consists of  $S$  consecutive stages of dating, with  $S < n$ . (In other words, the length of adolescence is  $S$ .) At stage  $s \in \{1, 2, \dots, S\}$ , individual  $i \in N$  randomly meets a date  $d(i, s)$  of opposite sex, whose mate value  $v(d(i, s))$  is immediately known to individual  $i$ . Individual  $i$  finds the date  $d(i, s)$  *desirable* at stage  $s$  if  $v(d(i, s)) \geq a(i, s - 1)$ , i.e., the mate value of the date is not below his/her aspiration at the beginning of stage  $s$ . Here, it is assumed that  $a(i, 0)$  is exogenously given to individual  $i$  at the beginning of stage 1. Depending on the adjustment rule, individual  $i$  and the date  $d(i, s)$  may exchange information as to whether they find each other desirable at stage  $s$ . Then, individual

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<sup>6</sup>See Conclusions for a discussion that the same question can be asked in evolutionary environments where individuals learn to play better search rules by mutations.

$i$  forms his/her aspiration level,  $a(i, s)$ , corresponding to stage  $s$ .

With the aspiration level  $a(i, S)$  formed at the last stage of the adolescence phase, individual  $i$  next enters the second phase of mate search, called the ‘mating’ phase. This phase may also have multiple stages depending on the mating outcome in the first stage. At the beginning of the first stage in the mating phase, all individuals are in the mating pool. In each stage of the mating phase, males and females in the mating pool are randomly paired to assess each other for a possible mating. If both individuals in a pair, after learning the mate values of each other, make a proposal to each other, then they are mated and removed from the mating pool. Otherwise, both individuals remain in the mating pool, as available for the next stage, if any. The mating phase ends after a finite stage at which either the mating pool becomes empty or each individual in the mating pool has already been paired with all available individuals of the opposite sex.

Below, we describe four adjustment rules (taken from Todd and Miller, 1999), according to which individuals can update their aspiration levels in the adolescence phase.

**Take the Next Best (TNB) Rule:** This is a modification of the 37% rule in the “secretary problem” (Ferguson, 1989; Seale and Rapoport, 1997), as each individual dates with  $(S/n)\%$  of the available candidate mates in the adolescence phase.<sup>7</sup> According to this rule, at a stage of dating  $s$ , individual  $i$  sets the corresponding

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<sup>7</sup>In the secretary problem, an employer must hire the best applicant for a secretarial job, interviewing each applicant one at a time without being able to make a job offer to an already interviewed applicant. The employer knows the number of applicants but does not know the distribution of the applicants. In this setup, the optimal strategy of the employer turns out to be first interviewing (approximately) %37 of the available applicants and choosing in the following hiring period the next better applicant whose quality is above the quality of the best applicant interviewed. The TNB Rule considered by Todd (1997, 1999) is similar to the search rule in the secretary problem except for that (i) the number of potential mates does not need to be known by any individual searching for a mate and (ii) individuals do not optimize but use *heuristics* they find to be satisficing.



aspiration level to the mate value of the date,  $v(d(i, s))$ , if individual  $i$  finds the date  $d(i, s)$  desirable, and sets it to the aspiration level corresponding to the previous stage,  $a(i, s - 1)$ , otherwise. Formally,

$$a(i, s) = \begin{cases} v(d(i, s)) & \text{if } v(d(i, s)) \geq a(i, s - 1), \\ a(i, s - 1) & \text{otherwise.} \end{cases}$$

So, individual  $i$  enters the mating phase with the aspiration level  $a(i, S) = \max\{a(i, 0), v(d(i, 1)), v(d(i, 2)), \dots, v(d(i, S))\}$ . For this rule,  $a(i, 0)$  is assumed to be zero, the lowest possible mate value.

For the following three adjustment rules, it is assumed that at each stage of learning each individual is informed by the date whether the date found him/her desirable. Moreover, for these adjustment rules  $a(i, 0)$  is assumed to be  $V/2$ , the mean mate value of all males and of all females.

**Adjust Up/Down Rule:** This rule is formulated as follows:

$$a(i, s) = \begin{cases} a(i, s - 1) + \bar{\delta} & \text{if } v(i) \geq a(d(i, s), s - 1), \\ a(i, s - 1) - \bar{\delta} & \text{otherwise,} \end{cases}$$

where  $\bar{\delta} = (n/2)/(1 + S)$ .

Here, individual  $i$  adjusts up his/her stage  $s - 1$  aspiration  $a(i, s - 1)$  by the constant  $\bar{\delta}$  to obtain stage  $s$  aspiration  $a(i, s)$  if the date  $d(i, s)$  finds individual  $i$  desirable. Otherwise, individual  $i$  adjusts down  $a(i, s - 1)$  by  $\bar{\delta}$  to obtain  $a(i, s)$ .

**Adjust Relative Rule:** According to this rule, the aspiration of individual  $i$  at stage  $s$  is given by

$$a(i, s) = \begin{cases} a(i, s - 1) + \bar{\delta} & \text{if } v(i) \geq a(d(i, s), s - 1) \text{ and } v(d(i, s)) \geq a(i, s - 1), \\ a(i, s - 1) - \bar{\delta} & \text{if } v(i) < a(d(i, s), s - 1) \text{ and } v(d(i, s)) < a(i, s - 1), \\ a(i, s - 1) & \text{otherwise,} \end{cases}$$

where  $\bar{\delta} = (n/2)/(1 + S)$ .

Differing from the previous rule, now there is the possibility of nonadjusting in addition to up and down adjusting. Here, individual  $i$  adjusts up his/her stage  $s - 1$  aspiration  $a(i, s - 1)$  by  $\bar{\delta}$  to obtain stage  $s$  aspiration  $a(i, s)$  if individual  $i$  and the date  $d(i, s)$  find each other desirable. If none of the dating individuals  $i$  and  $d(i, s)$  finds the other desirable, then individual  $i$  adjusts down  $a(i, s - 1)$  by  $\bar{\delta}$  to obtain  $a(i, s)$ . In other possible cases, individual  $i$  does not adjust his/her aspiration level at stage  $s$  and he/she sets  $a(i, s)$  to  $a(i, s - 1)$ .

**Adjust Relative/2 Rule:** This rule differs from the Adjust Relative Rule in that the size of adjustments is neither constant over the individuals nor over the stages of adolescence. For individual  $i$ , the size of adjustment at stage  $s$  is equal to the half of the difference between the mate value of the date and the aspiration level of individual  $i$  at the end of previous stage. Thus, the rule is given by

$$a(i, s) = \begin{cases} a(i, s - 1) + \delta(i, s) & \text{if } v(i) \geq a(d(i, s), s - 1) \text{ and } v(d(i, s)) \geq a(i, s - 1), \\ a(i, s - 1) - \delta(i, s) & \text{if } v(i) < a(d(i, s), s - 1) \text{ and } v(d(i, s)) < a(i, s - 1), \\ a(i, s - 1) & \text{otherwise,} \end{cases}$$

where  $\delta(i, s) = |v(d(i, s)) - a(i, s - 1)|/2$ .

Using a population with  $n = 100$  (i.e., 100 males and 100 females), the maximal mate value  $V$  set to 100, and mate values uniformly distributed to individuals, Todd and Miller (1999) simulated the likelihood of mating (the number of mated pairs formed) corresponding to each adjustment rule, when the length of adolescence  $S$  is changed from 1 to 90. Considering the same mating environment, we have conducted 200 (Monte Carlo) simulations at each value of  $S$  to reproduce their findings in Figure 1. (We have used the GAUSS software for all simulations in this paper. The program codes and the simulated data are available from the author upon request.)

Apparently, for all considered adolescence lengths, the TNB rule is dominated

by each of the other rules in terms of the produced likelihood of mating. Figure 1 also shows two sharp findings that for short adolescence lengths ( $2 \leq S \leq 32$ ), the Adjust Relative/2 Rule generates the highest number of matings among the four adjustment rules; whereas when adolescence is medium to long ( $42 \leq S \leq 90$ ), the Adjust Up/Down Rule generates the highest number of matings, performing slightly better than the Adjust Relative Rule. It is also evident that both Adjust Up/Down and Adjust Relative are significantly superior to Adjust Relative/2 when adolescence is long.

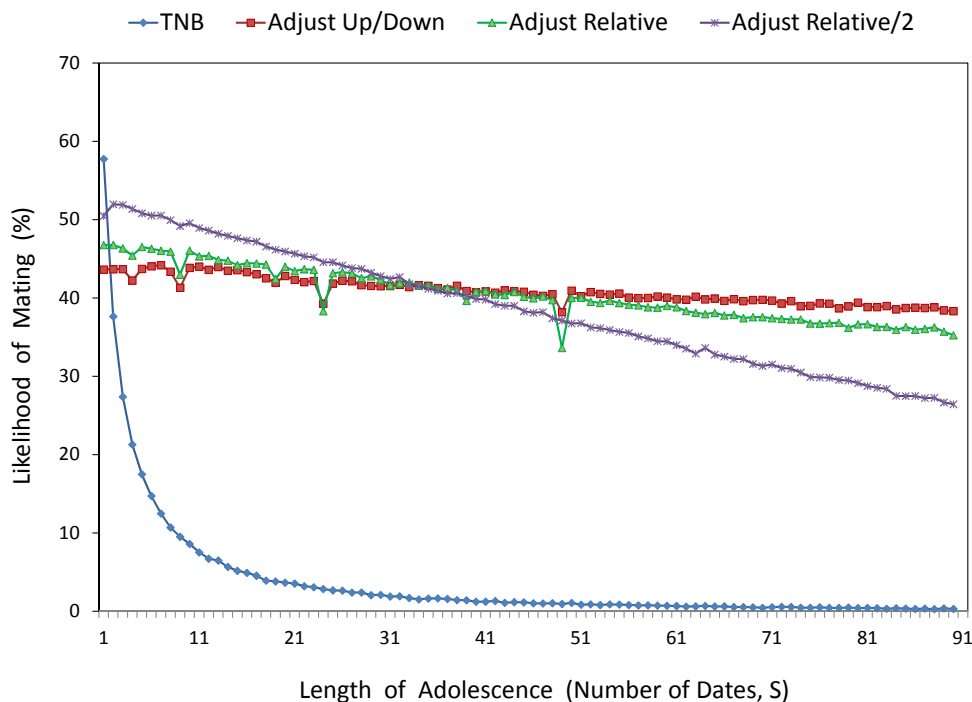


Figure 1. The number of successful mates

### 3 Stability of Adjustment Rules

We will check whether all individuals using a particular adjustment rule can be a Nash equilibrium of a normal-form strategic game played right before the adolescence

phase. In this game, the set of players involve all individuals in the population, the strategies of the players are restricted to the four adjustment rules we have described above and the payoff of each player at each strategy profile is simply his/her likelihood of mating. For a formal treatment, we introduce the following definitions.

Let  $\mathcal{R} = \{\text{TNB}, \text{Adjust Up/Down}, \text{Adjust Relative}, \text{Adjust Relative}/2\}$  denote the strategy space of each individual  $i$ . Let  $r = (r_{m_1}, \dots, r_{m_n}, r_{f_1}, \dots, r_{f_n}) \in \mathcal{R}^{2n}$  denote the strategy profile of the society. In particular, we denote by  $r^{TNB}$  the strategy profile at which each individual plays the strategy TNB, i.e.  $r_i^{TNB} = \text{TNB}$  for all  $i \in N$ . We similarly define the strategy profiles  $r^{AUD}$ ,  $r^{AR}$ , and  $r^{AR2}$  such that all individuals in the society play Adjust Up/Down under the profile  $r^{AUD}$ , Adjust Relative under  $r^{AR}$ , and Adjust Relative/2 under  $r^{AR2}$ . Also, for all  $i$  and  $r \in \mathcal{R}^{2n}$  define the  $2n - 1$  dimensional profile  $r_{-i}$  such that  $r = (r_i, r_{-i})$ . For any strategy profile  $r \in \mathcal{R}^{2n}$ , let  $\mu_i(r)$  denote the likelihood that individual  $i$  is mated to someone of the opposite sex when he/she uses the strategy  $r_i$ , while the rest of the society uses their respective strategies in  $r_{-i}$ .

We say that a strategy profile  $r$  is a Nash equilibrium if there exists no individual that can increase his/her likelihood of mating by unilaterally deviating from this profile by changing his/her strategy  $r_i$  to any other strategy  $r'_i$  in  $\mathcal{R}$ ; i.e., the profile  $r = (r_i, r_{-i}) \in \mathcal{R}^{2n}$  is a Nash equilibrium if

$$\mu_i(r_i, r_{-i}) \geq \mu_i(r'_i, r_{-i}) \quad \text{for all } i \text{ and for all } r'_i \in \mathcal{R}.$$

Below, we explore whether any of the profiles  $r^{TNB}$ ,  $r^{AUD}$ ,  $r^{AR}$ , and  $r^{AR2}$  is a Nash equilibrium for any length of adolescence. For each of these profiles, we make 200 Monte Carlo simulations at each value of  $S$  between 1 and 90. At each simulation, we randomly pick one of the individuals (i.e., %1 of 100 individuals of a particular sex) to be a potential deviant and check whether this individual can increase his/her likelihood of mating by unilaterally switching from the population's common strategy to any other strategy in  $\mathcal{R}$ . (Since the model is completely symmetric with respect to all individuals, checking whether or not an arbitrarily chosen individual has an

incentive to deviate is sufficient for our purpose.)

We check in Figure 2 whether  $r^{TNB}$  is a Nash equilibrium profile for any length of adolescence. To that end, we simply compare the values of  $\mu_i(r^{TNB})$  (in blue marked points), denoting the likelihood individual  $i$  is mated when he/she sticks to the common strategy  $TNB$  of the rest of the population, with the values of  $\max_{r'_i \in \mathcal{R} \setminus \{TNB\}} \mu_i(r'_i, r_{-i}^{TNB})$  (in red marked points), denoting the likelihood individual  $i$  is mated when he/she deviates to the best alternative strategy in  $\mathcal{R} \setminus \{TNB\}$ .

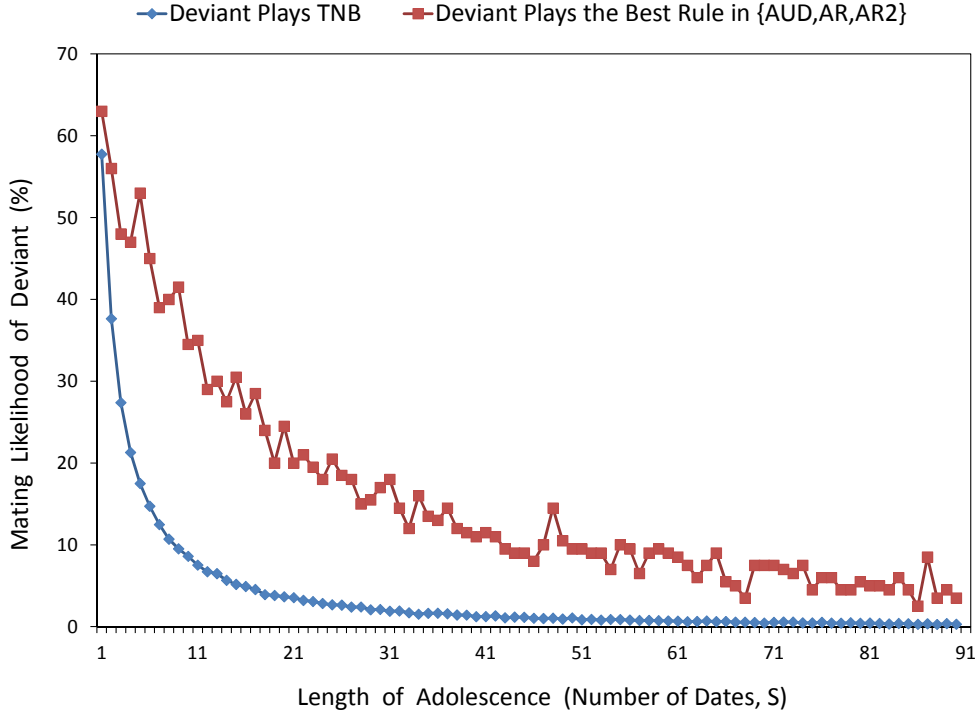


Figure 2. The mating likelihood of a potential deviant when he/she plays TNB versus the best alternative rule, while the rest of the society plays TNB.

It is apparent in Figure 2 that  $r^{TNB}$  is not a Nash equilibrium profile for any value of  $S = 1, 2, \dots, 90$ . This result is not surprising since unlike the other rules TNB makes adjustments always in the upward direction and does not depend on whether

the potential deviant is found desirable by his/her date. Therefore, the aspiration level of a potential deviant in the mating phase is higher under TNB than under the alternative adjustment rules. Since the lower the aspiration level of an individual, the more likely he/she will accept a proposal in the mating period, an individual can increase his/her likelihood of mating by switching from TNB to the best alternative strategy in  $\mathcal{R}$ .

In Figure 3 we show that  $r^{AUD}$  is not a Nash equilibrium profile for short to medium lengths of adolescence ( $S < 46$ ). For higher lengths of adolescence,  $r^{AUD}$  may turn out to be a Nash equilibrium profile. Yet, this is only true for 12 out of all values of  $S$  between 46 and 90.

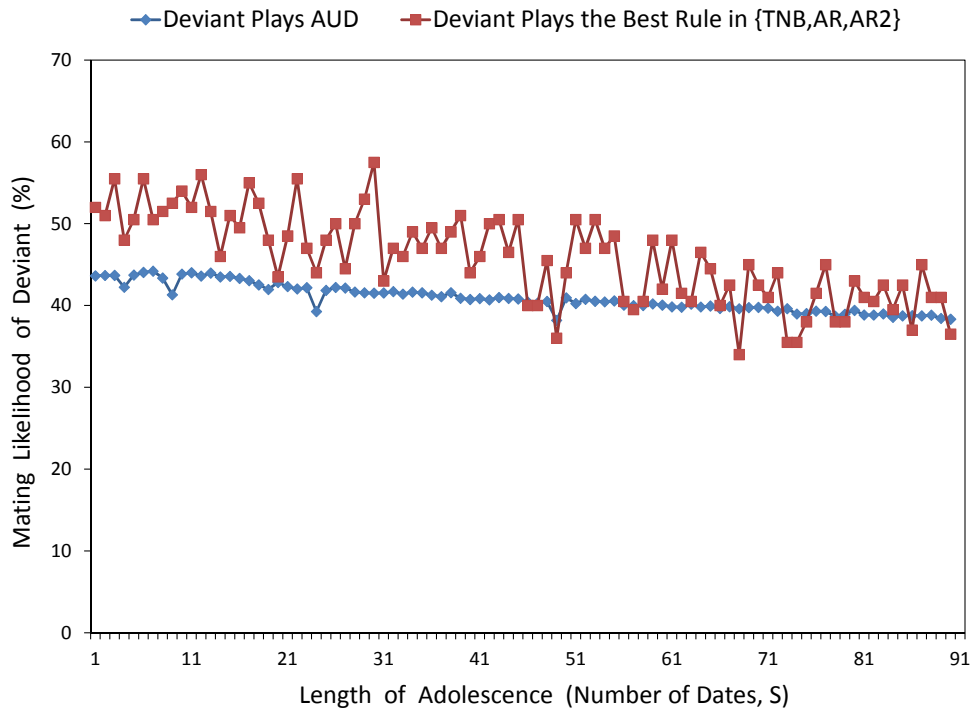


Figure 3. The mating likelihood of a potential deviant when he/she plays AUD versus the best alternative rule, when the rest of the society plays AUD.

A closer inspection of the simulation data generating Figure 3 also reveals that

a randomly selected individual prefers to play Adjust Relative at 65 values of  $S$  and Adjust Relative/2 at only 13 values of  $S$ , out of a total of 78 distinct values of  $S$  at which he/she finds it optimal to deviate from the Adjust Up/Down Rule.

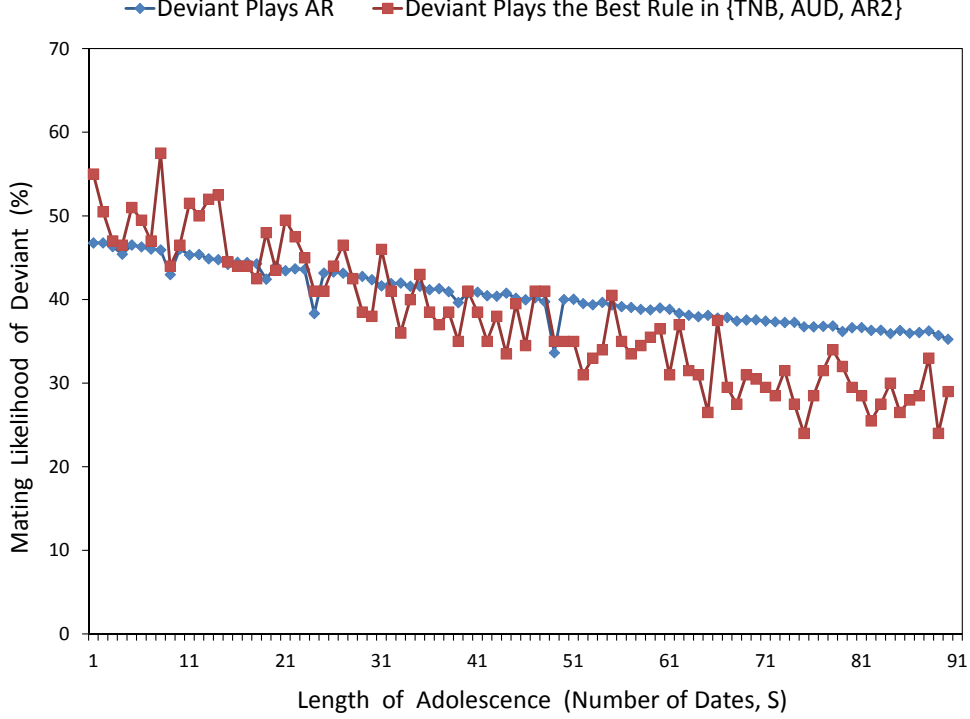


Figure 4. The mating likelihood of a potential deviant when he/she plays AR versus the best alternative rule, when the rest of the society plays AR.

In Figure 4, we illustrate that  $r^{AR}$  is a Nash equilibrium profile for most of the medium to high lengths of adolescence (i.e., for all values of  $S$  exceeding 55 and for 22 values of  $S$  between 28 and 55). whereas for most of the short lengths of adolescence (i.e., for 22 out of the lowest 27 values of  $S$ ) an arbitrary individual has an incentive to unilaterally deviate from playing Adjust Relative. Only at four out of 28 values  $S$  where  $r^{AR}$  is not found to be a Nash equilibrium, the deviating individual prefers to play Adjust Up/Down, while he/she plays Adjust Relative/2 in the remaining 24 instances.

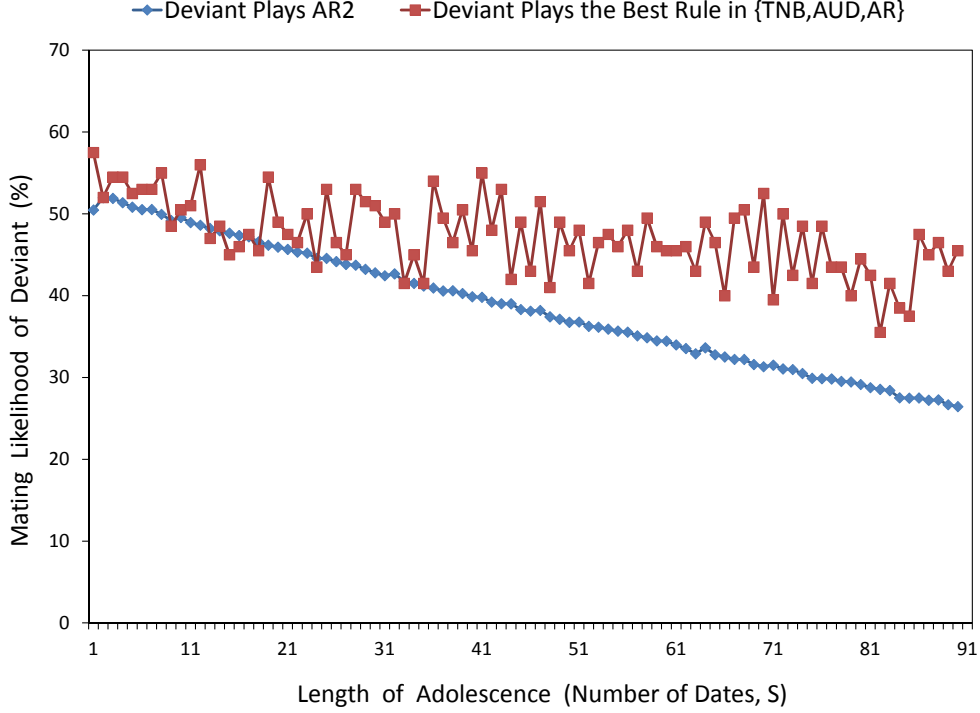


Figure 5. The mating likelihood of a potential deviant when he/she plays AR2 versus the best alternative rule, when the rest of the society plays AR2.

Finally, in Figure 5 we plot the equilibrium results for Adjust Relative/2. Here, we find that  $r^{AR2}$  is not a Nash equilibrium profile for any value of  $S$  exceeding 33. In the view of the potential deviant, playing alternative adjustment rules rather than Adjust Relative/2 seems to be attractive for shorter lengths of adolescence, as well; indeed we find that only for 7 of the lowest 33 values of  $S$ , the profile  $r^{AR2}$  can arise as a Nash equilibrium. In more detail, the deviating individual prefers to play Adjust Relative in 68 out of 83 values of  $S$  at which  $r^{AR2}$  is not a Nash equilibrium, whereas he/she plays Adjust Up/Down in the remaining 15 cases.

From the above results, we immediately notice that for a majority of adolescence lengths, the Adjust Relative Rule is the best strategy of a deviant at both of the profiles  $r^{AUD}$  and  $r^{AR2}$ . This is so, despite the observation in Figure 1 that Adjust



Up/Down dominates both the Adjust Relative and Adjust Relative/2 Rules, in terms of the induced likelihood of mating, for more than half of the considered values of  $S$  (i.e., for  $42 \leq S \leq 90$ ) while the Adjust Relative and Adjust Relative/2 Rules generate quite similar outcomes on the average over the whole simulation range. Interestingly, these two rules that are inferior to Adjust Up/Down for a majority of adolescence lengths when they are played by the whole population can become superior to it for some adolescence lengths when they are played singly by any individual. A possible explanation underlying this phenomenon may be related to the variance of aspirations generated by these rules. We observe that although the rules, Adjust Up/Down, Adjust Relative, and Adjust Relative/2, have almost the same mean aspiration level, close to the mean mate value of 50, for almost all adolescence lengths, the standard deviation of aspiration around the mean value is significantly different for these rules, as reported below for a sample of values of  $S$ .

Table 1. The mean value of the standard deviation of aspiration levels under the rules AUD, AR, and AR2

S	AUD	AR	AR2
10	34,01	16,55	24,16
30	34,64	16,62	27,95
50	34,69	16,94	28,34
70	34,92	16,92	28,68
90	34,76	16,85	28,80

In the above table, the standard deviation of aspirations is lower under the rules Adjust Relative (around 16) and Adjust Relative/2 (between 24-29) than under Adjust Up/Down (around 34) since not only that the former rules allow the possibility of nonadjusting the aspiration after a date, but also the conditions for adjusting it are stronger (as the mutual desirability of the dating partners is required). On the

other hand, the reason why Adjust Relative and Adjust Relative/2 themselves yield significantly different standard deviations of aspirations should be the difference of the size of adjustments in the definition of these two rules. Considering our findings, it seems that the smaller the variance of aspirations generated by a particular adjustment rule, the more likely its survival against the alternative strategies of potential deviants, yet a formal proof of the relation between the stability of a rule and the induced aspirations in agent-based models is left to future research.

## 4 Conclusions

In this paper, we have studied whether the use of a particular heuristic by the whole population in mutual sequential mate search can be a stable situation, where no individual has any incentive to use an alternative heuristic in order to increase his/her likelihood of mating. For our purpose, we have considered the Nash equilibrium as a proper concept of stability, and have restricted our focus on mate search heuristics to four rules, namely TNB, Adjust Up/Down, Adjust Relative, and Adjust Relative/2, that were considered by Todd and Miller (1999). Using a two-phase search model of theirs, which involves an adolescence phase and a mating phase, we have showed that in the whole simulation range of adolescence lengths, the unanimous use of the TNB Rule by the whole population never arises as an equilibrium of a strategic game played right before the adolescence phase. Of the other three rules, Adjust Up/Down and Adjust Relative/2 have been observed as a Nash equilibrium play, though only for a small part of the simulation range; the former arising when the adolescence is long and the latter arising when the adolescence is short. On the other side, the Adjust Relative Rule has been an equilibrium strategy for the whole population for a great part of the simulation range, especially for medium to high adolescence lengths. Taking stock of our results, the stability of heuristics as a new measure of mating success points to that among the mate search rules we have considered, the Adjust Relative Rule appears to be the one which is most likely to survive in

strategic environments.

We believe that the contribution of this study to the previous literature on mate search is at least twofold. First, we add some new results to a very thin literature dealing with the stability issues in agent-based search models, hoping to narrow down the existing gap between the priorities of stable matching theory and agent-based mate search. However, unlike the previous works (Eriksson and Hägsström, 2008; Eriksson and Strimling, 2009), our focus is on the stability of mate search heuristics, using the Nash equilibrium concept, instead of the stability of matching outcome in the usual definitions of blocking individuals or pairs, since the latter can be a relevant measure of success to distinguish between alternative search heuristics in mate search models only if these heuristics are themselves stable in the long-run with respect to the invasion of alternative heuristics. Second, since a particular search heuristic can be stable, or form a Nash equilibrium, only if no individual in the population has any incentive to unilaterally switch to an alternative heuristic, our results show the robustness of some of the search heuristics considered by Todd and Miller (1997, 1999) with respect to the assumption that the whole population uses the same particular heuristic during the mate search.

One potential criticism to our study, as it deals with the stability of simple heuristical search rules in a model with assumedly non-optimizing agents, could be that we restrict our stability notion, for the calculational simplicity, to an ‘intelligent’ concept such as Nash equilibrium, which requires that the beliefs of each individual about what strategies are likely to be played by other individuals are common knowledge and also that each individual is endowed with the skill of performing optimization over the outcomes of alternative strategies. However, our appeal to the Nash equilibrium concept is not wholly illegitimate since an observation that the play of a particular heuristic by the whole population is not a Nash equilibrium would directly point to the existence of a better heuristic from the viewpoint of a unilaterally deviating individual. Clearly, such an individual could find the merit of playing this alternative heuristic also under the notion of evolutionary stable strate-

gies introduced by the evolutionary model of Maynard Smith and Price (1973) and Maynard Smith (1974), which involve non-optimizing individuals (players) some of whom may deviate from a particular common (incumbent) rule only because they are programmed to do so or alternatively by ‘simple’ or ‘unsophisticated’ reasons, involving mistakes, ignorance, etc, simply called ‘mutations’.<sup>8</sup>

Finally, we should notice that the possibility of unilateral deviations of individuals - under the stability concept we have considered - from a particular search rule commonly used by the whole population to another search rule naturally brings in a broader question as to why the mate search model does not set, in the first place, each individual in the mating population entirely free, in using any available search rule during the mate search, independent from the set of rules used by other individuals. Constructing such a heterogenous model of mate search, the future research may profitably deal with finding the efficient distributions of search rules among the individuals in a given mating population (or equivalently, the optimal asymmetry level in the model) that will optimize a particular measure of mating success. Using the approach in this paper, one could also search for stable distributions of search rules, and in particular the stable distributions among the efficient ones.

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<sup>8</sup>In the evolutionary model of Maynard Smith and Price (1973) and Maynard Smith (1974), an incumbent strategy is called *evolutionary stable* if for each mutant strategy there exists a threshold such that the incumbent strategy yields a higher fitness (payoff) than the mutant strategy when the share of individuals playing the mutant strategy is below this threshold. Since each evolutionary stable strategy must be trivially optimal against itself, the set of evolutionary stable strategies are always contained by the set of Nash equilibria.

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